THE GROUP METHOD OF DATA HANDLING AND ANALYTIC HIERARCHY PROCESS IN DECISION MAKING PROBLEMS

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ABSTRACT

Nowadays Saaty's Analytical Hierarchy Process (AHP) is the most applicable one for solving practical problems of multicriteria choice. In case the alternatives in AHP are vectors the problem of multicriteria choice can be also solved with the help of the Method of Group Accounting of Arguments by Ivahnenko.
The Method of Group Accounting of Arguments and Hierarchy Analysis that combines the advantages and peculiarities of both methods mentioned above is proposed in the article.

Keywords: Analytic Hierarchy Process, pairwise comparison matrix, evaluation function, the Group Method of Data Handling, particular description, regularity criterion, learning subsample, test subsample.

1 INTRODUCTION

The decision making process is one of the most important components in every fields of human activity. In many cases the result of a single decision can have a total influence on the growth of business of the company. That's why a great attention is paid to development and research of effective methods of decision making problems solving.
While making operating decision and forecasting potential results, making decision person usually deals with sophisticated system of interconnected components (sources, desirable goals, etc.) that should be analyzed.

2 CASE I

If it is necessary to find the best alternative of the alternative set $\left\{A_i, i=\overline{1,m}\right\}$ from global goal point of view Analytic Hierarchy Process (AHP) by Saaty can be used [1-4]. Global goal can be set quantitatively by numerical scalar limited continuous function $f(A)$, where A – is free alternative that is defined by n-dimensional vector $\bar{x}=(x_1,...,x_n)^T$, where x_j, $j=\overline{1,n}$ - limited numerical value of j index, that characterizes alternative A.
Identity and decomposition principle of AHP provides for problem structuring as a network or hierarchy.[5] (Figure 1)

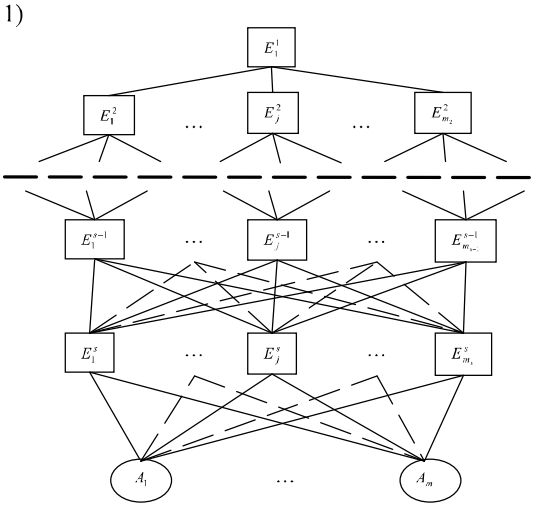

Figure 1. Hierarchical representation of making decision problem example

There are *m* alternatives $A_1,...,A_m$ and *s* criteria levels $E_j^i,\ i=\overline{1,s}\ \ j=\overline{1,m_i}$ in this problem (Figure 1).

It is necessary to find weights $w_1,\mathrm{K}\ ,w_n$ of the objects $A_1,\mathrm{K}\ ,A_n$ (alternatives, criteria) according to the global goal. Weights can be found by using empirical pairwise comparison matrix, that is provided by the experts. If pairwise comparison matrix is well matched, then $\gamma_{ij}=\frac{w_i}{w_j}$, where w_i is the weight of alternative *i*, w_j is the weight of alternative *j*.

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For this case, according to Saaty components of pairwise comparison matrix eigenvector, that corresponds to maximal eigenvalue $\lambda_{\max}$, can be used as weights $w_j, j=\overline{1,n}$. Unfortunately, real pairwise comparison matrixes usually are not well matched because of the object $A_1,\mathrm{K}\ ,A_n$ possible contradictoriness of properties manifestation and because of the different physiological factors that make influence on the experts. That's why the problem of weights finding by partially matched and unmatched matrixes appears.

Optimization models providing object weight finding with high precision by partially matched and unmatched matrixes have been developed by professor Pavlov [6-9]. We also have to emphasize that this models suit for object weights finding in multicriterion choice problems in which making decision person should analyze a great number of alternatives.

3 CASE II

If the problem of building of evaluation function $\hat{f}(A)=\hat{f}(x_1,\mathrm{K}\ ,x_n)$ of unknown function $f(A)=f(x_1,\mathrm{K}\ ,x_n)$ is defined it is appropriate to use the Group Method of Data Handling [10-11]. The experts specify the values $f(A_i),\ i=\overline{1,m}$ of unknown goal function on alternatives $A_i,\ i=\overline{1,m}$ taking additional data into consideration.

So, the problems mentioned above can be solved with the engagement of the experts. However, in practice it is more efficiently to limit the participation of the experts in problem solving that can reduce time and financial expenses. For this purpose the method, that combines the possibilities of Group Method of Data Handling and of Analytic Hierarchy Process and having the advantages of both ones can be proposed. This method is called by the authors "The Group Method of Data Handling and Analytic Hierarchy Process" (GMDH&AHP) [12].

Application of this method permits to build the function that approximates qualitative specified global goal. Thus, the process of the best alternative choice of the free set of alternatives without engagement of experts and, that is, without additional time and financial expenses is automated.

4 PROBLEM DEFINITION

It is necessary to restore effective evaluation $\hat{f}(A)$ of the unknown function $f(A)$ reflecting the global goal. As a rule, expert can't directly measure values $f(A_i), i=\overline{1,m}.$ But it is naturally to assume that expert can define the set of inequalities $G\{(A_i \succ A_j)\}\ \ i\neq j,(ij)\in I\ ;\ i,j\in\{\overline{1,m}\}$. The expert defines each inequality when he is sure that alternative A_i is better than the alternative A_j from global goal point of view.

Inequalities that belong to the set *G* mustn't break the condition of transitivity. If they do, numerical scalar limited continuous function $f(A)$ doesn't exist and the problem has no sense.

5 THE IDEA OF THE GROUP METHOD OF DATA HANDLING AND ANALYTIC HIERARCHY PROCESS

The idea of the method is the following. At first Analytic Hierarchy Process by Saaty has to be developed. $A_{i1},\mathrm{K}\ ,A_{ip}$ - the alternatives defined by the vectors $\overline{x_{i1}},...,\overline{x_{ip}}$ present the lower row of hierarchical tree. The

values of estimate $f\left(A_{ij}\right)=f\left(\overline{x_{ij}}\right),\ j=\overline{1,p}$ have to be found with the help of AHP. The problem is to restore numerical scalar continuous function $f\left(\overline{x}\right)=f\left(x_1,\text{K},x_n\right)$.

The Group Method of Data Handling is used in the following modification.

1) The values $f\left(\overline{x_{ij}}\right),\ j=\overline{1,p}$, received with the help of AHP present the learning subsample of data.

2) The set G that can be interpreted as the set of inequalities $\left(f\left(\overline{x_i}\right)>f\left(\overline{x_j}\right)\ \text{or}\ f\left(\overline{x_i}\right)\geq f\left(\overline{x_j}\right)\ \text{or}\ f\left(\overline{x_i}\right)=f\left(\overline{x_j}\right)\right)\ i\neq j;(ij)\in I;i,j=\left\{\overline{1,m}\right\}$. (if the condition of transitivity isn't broken) presents the test subsample of data. Then any criterion of particular description choice from the previous row of GMDH to the next one is based on the analysis of the expressions in dependence of the type of element $(ij)\in I$ or A_i f A_j; or A_i f A_j; or $A_i\sim A_j$;

$M_{kp}^{(ij)}=\left\{0,\ \text{if}\ f^{kp}\left(\overline{x_i}\right)>f^{kp}\left(\overline{x_j}\right)\ \text{or}\ f^{kp}\left(\overline{x}\right)\geq f^{kp}\left(x_j\right)\ \text{or}\ f^{kp}\left(x_i\right)=f^{kp}\left(x_j\right)\ \text{depending on the type of the}\right.$

element $(ij)\in I;\ \left|f^{kp}\left(\overline{x_i}\right)-f^{kp}\left(\overline{x_j}\right)\right|\ \left.\text{in the opposite case}\right\}$

$(\ ij\ )\in I;\ \kappa-$ is the number of the row of GMDH, p - is the number of particular description on the k-th row of GMDH. $f^{kp}\left(\overline{x}\right)$ is the p-th particular description on the k-th row of GMDH. For example, total estimate of breach of constraints of the set *G* can be introduced.

$M_2^{kp}=\frac{1}{|I|}\sum_{(ij)\in I}M_{kp}^{(ij)}$, $|I|$ - the number of elements in I. We have to let pass defined number of particular descriptions of the previous level with the lowest values M_2^{kp}.

3) Any GMDH regularity criterion can be used as the stopping criterion. Taking into consideration the specificity of the problem under discussion the following regularity criteria can be proposed:

a) M_1^{kp} - the measure of divergence of p-th particular description on the k-th row of GMDH, that was computed by the learning subsample $\frac{1}{p}\sum_{j=1}^{p}\left|f\left(\overline{x_{ij}}\right)-f^{kp}\left(\overline{x_{ij}}\right)\right|$.

Then, the decision of the problem is the particular description of GMDH hierarchy level with the lowest number for which $M_{kp}^1=M_{kp}^2=0$ is reached.

b) The decision of the problem is the particular description of GMDH hierarchy level with the lowest number for which $\min_p M_1^{kp}$ is reached under the condition $M_2^{kp}=0$.

c) The decision of the problem is the particular description of GMDH hierarchy level with the lowest number for which local minimum of the function $\min_p\left\{M_1^{kp}+M_2^{kp}\right\}$ is reached.

6 CONCLUSIONS

Application of the Group Method of Data Handling and Analytic Hierarchy Process permits to build the function that approximates qualitative specified global goal and gives the opportunity to automate the process of the best alternative choice of the free set of alternatives.

7 REFERENCES

1) Saaty T.L. Multicriteria Decision Making. The Analytic Hierarchy Process,. – New York: McGraw Hill International, 1990.- p.437.

2) Saaty T.L. The Analytic Network Process. –Pittsburgh: RWS Publications, 2001.-p. 386.

3) Саати Т. Принятие решений. Метод анализа иерархий: Tomas Saaty. The Analytic Hierarchy Process. –Пер. с англ. Р.Г. Вачнадзе. - М.: Радио и связь, 1993. – 315 с.

4) Саати Т., Кернс К. Аналитическое планирование. Организация систем: Пер. с англ. Р.Г. Вачнадзе: Под ред. И.А. Ушакова. - М.: Радио и связь, 1991. – 223 с.

5)Андрейчиков А.В. & Андрейчикова О.Н. Анализ, синтез, планирование решений в экономике. – Москва: Финансы и статистика. – 2001. – 342с.

6) Павлов А.А., Лищук Е.И. & Кут В.И. Математические модели оптимизации для обоснования и нахождения весов в методе парных сравнений. Системні дослідження та інформаційні технології 2007р. №2.

7) Павлов А.А., Лищук Е.И. & Кут В.И. Математические модели оптимизации для обоснования и нахождения весов объектов по неоднородным матрицам парных сравнений. Системні дослідження та інформаційні технології 2007р. №3

8) Павлов А.А., Лищук Е.И. & Кут В.Н. Многокритериальный выбор в задаче обработки данных матрицы парных сравнений. Вісник НТУУ „КПІ" Інформатика, управління та обчислювальна техніка, Київ 2007р. №46.

9) Павлов О.А., Іванова Г.А., Кут В.І. & Штанькевич О.С., Математичні моделі оптимізації для пошуку ваг об'єктів у методах підтримки прийняття рішень // Интеллектуальный анализ информации ИАИ-2008: Материалы VIII Международной конференции (14-17 мая 2008г., г.Киев) – К.: Просвіта, 2008. -608с.: ил. – С.361-366

10) Ивахненко А.Г. Долгосрочное прогнозирование и управление сложными системами. - Киев: Техника, 1975. - 312 с.

11) Ивахненко А.Г. & Юрачковский Ю.В. Моделирование сложных систем по экспериментальным данным. - М.: Радио и связь, 1986. -118с.

12) Павлов А.А., Иванова А.А. & Зигура. Р.А. Метод группового учёта аргументов и анализа иерархий (МГУАиАИ) в задачах принятия решений. Вісник НТУУ „КПІ" Інформатика, управління та обчислювальна техніка, Київ 2007р. №47.-350 с. – С.205-214.